The 9th International Conference "ENVIRONMENTAL ENGINEERING"
22–23 May 2014, Vilnius, Lithuania
SELECTED PAPERS
eISSN 2029-7092 / eISBN 978-609-457-640-9
Available online at *http://enviro.vgtu.lt*

Section: Technologies of Geodesy and Cadastre

Robust adjustment of vertical crustal movements in Poland, based on GNSS observations

Jacek Rapinski

Institute of Geodesy, University of Warmia and Mazury in Olsztyn, Poland

Abstract

Existence of permanent GNSS observation station provides the unseen earlier amount of data. This data can be used to calculate vertical crustal movements. In the presented paper, the adjustment of vertical movements network for the territory of Poland is presented. Both standard (least squares) and robust approaches are presented. For the computations author's own software was developed using C++ with Qt (for user interface) and Eigen (for matrix manipulations) libraries. Results are presented in the form of plots.

Keywords: verical crustal movements; GNSS leveling; robust estimation.

1. Introduction

Vertical crustal movements are usually calculated on the basis of precise leveling data. Availability of continuously operating reference stations gives the opportunity to calculate these movements on the basis of GNSS observations. Height differences between stations can be calculated for many epochs (daily, weekly or monthly). It gives a very large number of data, some of which can be contaminated with outliers. Minimizing the influence of outliers on results of adjustment is crucial to obtain a reliable vertical crustal movements of a network of points. On the basis of such network a vertical crustal movements map can be created [4]. In the following sections an attempt to verify the data from the perspective of outliers.

2. Data processing

2.1. Input data

Height differences between selected polish ASG-Eupos CORS were used as input data. The height differences were calculated from one day long observation sessions using Bernese 5.0 software at the Centre of Applied Geomatics, Military University of Technology. Connections between stations were determined using a Delanua'y triangulation. Daily height differences between stations will be referenced as observations. Since not all of the ASG-Eupos stations are operational for the same time, there are differences in the number of observations for each connection. Detailed description of how this data was processed is in section 2.2. Figure 1 depicts a time series of such data (between 0014-ZARY stations). The number of observations for each connection varies from 646 to 1778.

From Figure 1 it is clear that the data contains a lot of noise. First few epochs are significantly more noisy then the rest of data. This is due to the problems at the beginning of CORS operation. This kind of problems were fixed by removing faulty epochs from data in preprocessing stage. To perform a least squares adjustment we need to be sure that the distribution of the noise is symmetrical (preferably close to normal distribution). For this purpose a quantile – quantile plots and histograms for each data set were created and visually inspected. Quantile – quantile plot is a graphical method for comparing two probability distributions by plotting their quantiles on two axis of the same plot [2]. This type of evaluation was used to asset leveling data in [3]. An example for one connection is presented in Figure 2. From this figures it is clear that the underlying distribution has heavier tails compared to those of a normal distribution. Also the histograms are almost symmetric with no major signs of skew.

Corresponding author: Jacek Rapinski. E-mail address: jacek.rapinski@uwm.edu.pl
http://dx.doi.org/10.3846/enviro.2014.240

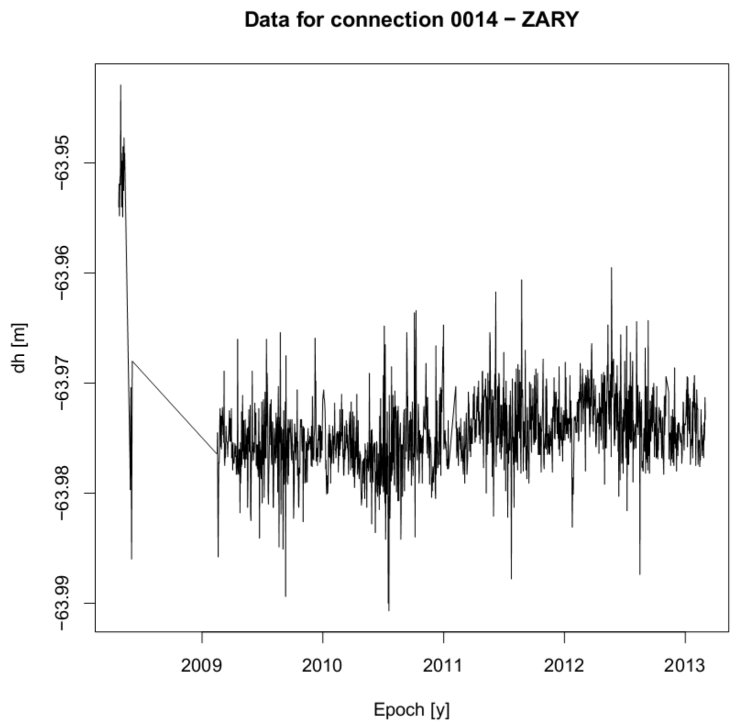


Fig. 1. Data for the 0014-ZARY connection

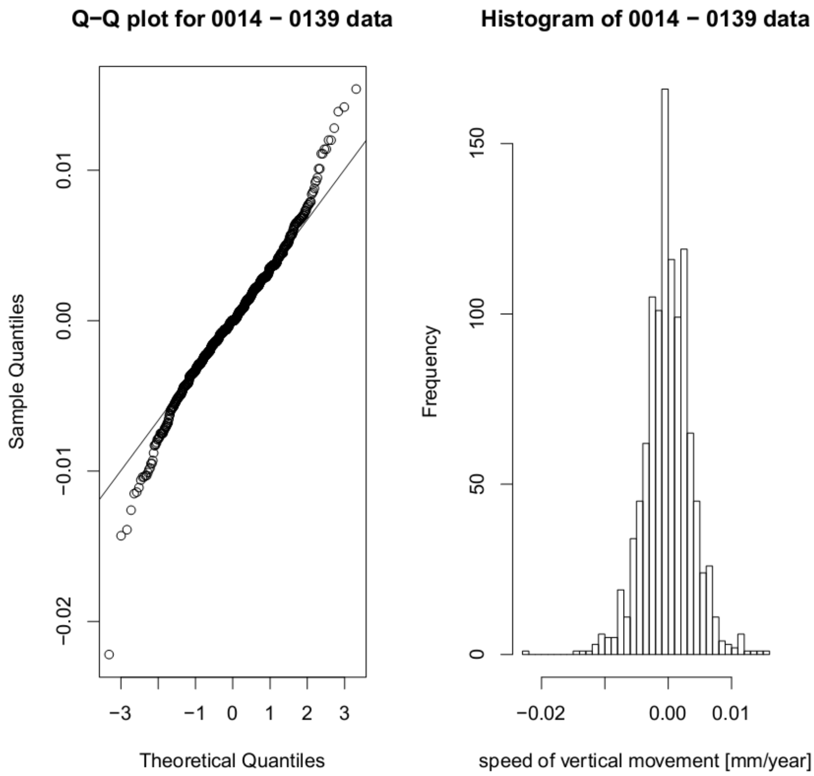


Fig. 2. Quantile – quantile plot and histogram of vertical movement of the example data

2.2. Adjustments

To perform adjustments and data analysis the vertAdj software was developed in C++ and QT framework on Ubuntu 13.04 platform. To perform Matrix operations the Eigen library (http://eigen.tuxfamily.org) was used. Developed software has the ability to calculate the movement for one connection in three ways:

- by avaraging the movements calculateds between consecutive epochs,
- by weighted least squares,
- by robust least squares.

2.2.1. Least squares

The standard approach would be to use a least squares method to determine the movement speed (which is a directional coefficient of line fitted into data). Since the number of data is not equal for each connection weighting of observations is introduced. The vertAdj software allows to choose if adjustment is using weights or not. If not a unit matrix is created as a weight matrix. If weighting is used weights are calculated using the number of observations for each connection.

$$P = \begin{bmatrix} n_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & n_m \end{bmatrix} \tag{1}$$

where ni is the number of observations for *i-th* connection.

2.2.2. Robust least squares

Robust adjustment is an adjustment in which the influence of outliers is minimized by appropriately modyfing the weight matrix. In robust least squares adjustment process is conducted in iterations. After each iteration weights for the next iteration are calculated using the equation (2) [1].

$$P_{i+1} = \frac{P_i}{2}\exp\left(\frac{-P_i V_i^2}{2}\right) \tag{2}$$

where P_i is a weight of particular observation in i-th iteration, and V_i is a residual. This approach attenuates stronger those weights which observations have largest residuals. Weights for observations with smaller residuals are attenuated less. Iterations ends when there is no significant increment of parameters or when the maximum number of iterations is reached

3. Data processing

In this paper data was processed in accordance to the diagram from Figure 3.

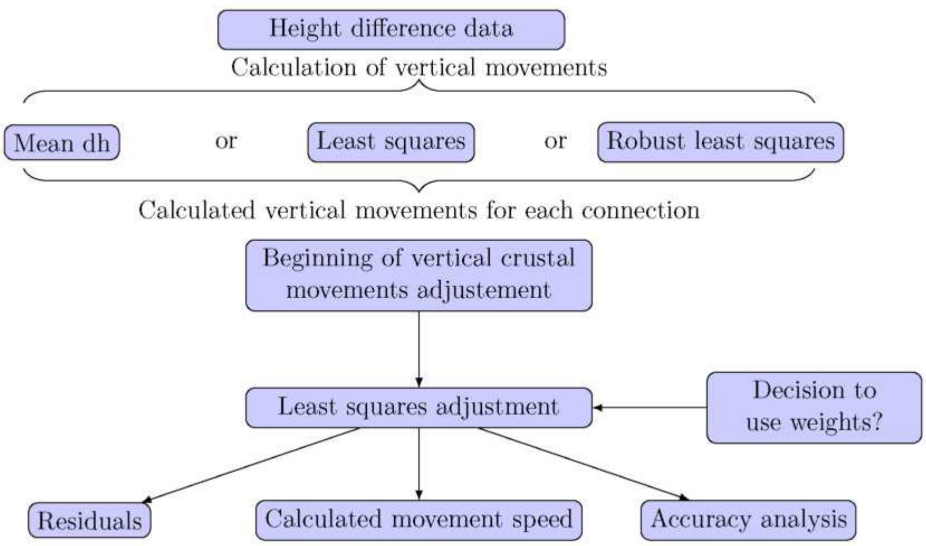


Fig. 3. Diagram of data processing

In the first step height differences data from daily observations for each connection was prepared. In the second step a relative vertical crustal movement for each connection was calculated. This was done in three scenarios:

– Movement calculated as a mean from movements between consecutive epochs

$$v = \frac{\sum_{i=1}^{n} \left(\frac{h_{i+1} - h_i}{\Delta t} \right)}{n-1} \tag{3}$$

where v is a movement speed, n is a number of height differences, At is a time span between two consecutive epochs, h_i is a height difference in i-th epoch.

– Movement calculated by the least squares method – fitting a line into a set of data:

$$h = vt + b \tag{4}$$

– Movement calculated using robust least squares – the same as above but with a robust estimation.

After this steps a single value of movement for each connection is available. This movements are then used as observations in vertical crustal movements network adjustment. As a results we obtain movements in each point, observation residuals and accuracy estimation.

4. Tests

To verify the usability of the robust estimation in data preparation process, some tests were performed.

4.1. Test area

The test area includes 31 stations resulting in 73 connections from the Sudety region. The network is depicted in Figure 4. Numbers over connections in Figure depicts a number of data for each connection.

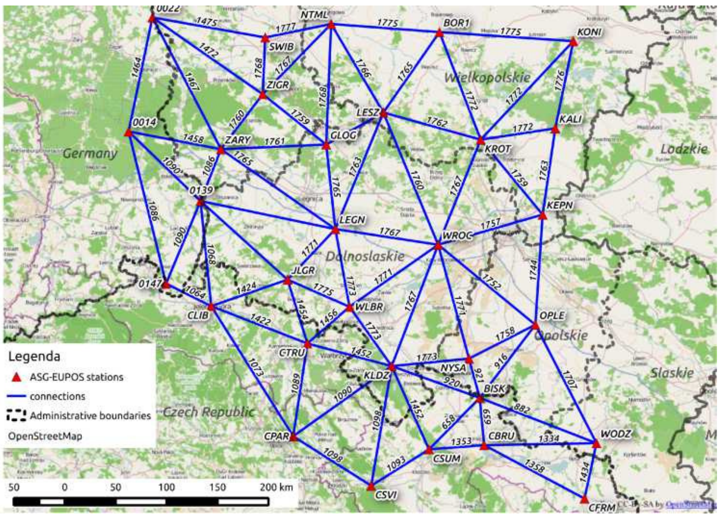
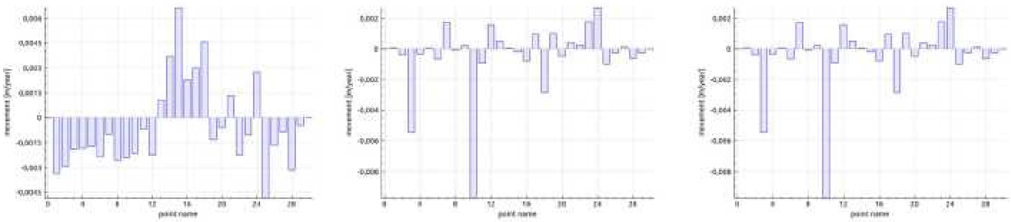


Fig. 4. Network used in the adjustment

For this network all three scenarios mentioned in section 3 were calculated. Resulting vertical crustal movements are depicted in Figure 5. Plots in Figure 5 depicts the calculated values of vertical crustal movements from the presented scenarios. From the Figure 5 it seems that there is almost no difference between least squares and robust least squares. The difference between simple averaging and remaining two methods is clear. Taking a closer look into residuals from least squares and robust least squares shows, that the differences between this two methods are very small when expressed in mm/year. An example for one connection is depicted in Figure 6.



(a) Scenario 1 – simple avarage (b) Scenario 2 – least squares (c) Scenario 3 – robust least squares

Fig. 5. Vertical crustal movements calculated using three scenarios

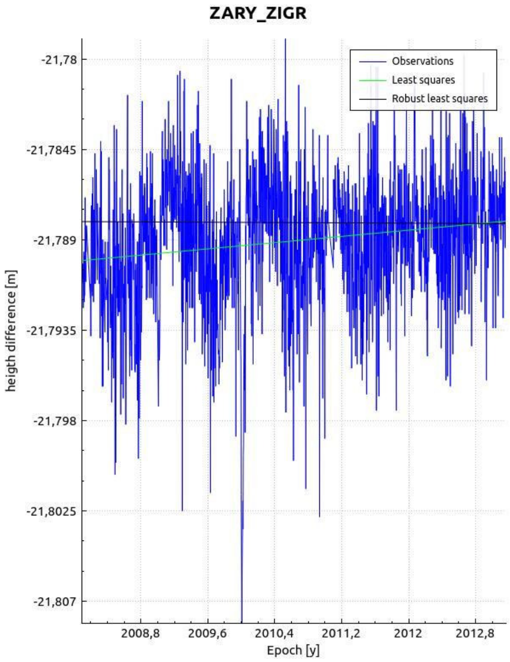


Fig. 6. Observations for one connection with lines fitted using least squares and robust least squares

5. Discussion

The results presented in previous section shows that the differences in vertical crustal movements for each connection resulting from the method of movement calculation is not significant for the adjustment of entire network. Results of movements for each point are almost the same for both methods (differences are order of magnitude smaller then measurement errors). It proves that there is no significant outliers in measurement data and that there is no need to use robust least squares in case of such elaboration.

References

[1] Kaminski, W.; Wisniewski, Z. 1992. Analiza wybranych, odpornych na bledy grube,metod wyrownania obserwacji geodezyjnych (in Polish), *Geodezja i Kartografi* 41(3–4): 173–181.
[2] Wilk, M. B.; Gnanadesikan, R. 1968. Probability plotting methods for the analysis for the analysis of data, *Biometrika* 55(1): 1–17.
[3] Kowalczyk, K.; Rapinsk, J. 2013. Evaluation of leveling data for use in vertical crustal movements model in Poland, Acta Geodyn. *Geomater* 10(3/171): 410–410.
[4] Kontny, B.; Bogusz, J. 2012. Models of vertical movements of the earth crust surface in the area of Poland derived from leveling and GNSS data, Acta Geodyn. *Geomater* 9(3/1167): 331–337.