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Determination of the point coordinates in 2D geodetic network solution using methods of reference and transition point indicators

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Abstract

A new algorithm for determining of the point coordinates in two-dimensional space in geodetic network solution is presented. Geodetic network solution should be understood as a system which consists of reference points, transition points, observations (distances) and the unknown coordinates of the point to be determined. The definition of a transition point indicator with regard to the determined point is introduced. Transition point indicator definition is based on the reference point indicator definitions developed by the author. With the use of definitions of these point indicators, there is a solution for the sought point in geodetic network solution. The direct solution without application of the least squares method is derived. In the proposed solution there is no need to know the initial approximate location of the determined point, nor the coordinates of the transition points. In this article the basic principles of the methods for solving the positioning problem in geodetic network are presented, and the formulas and their derivation are given. The numerical example with simulated data and proof confirm the correct performance of the proposed algorithm. The presented method should be tested with real measurements in many areas of positioning and navigation as well.

Keywords: Network solution; linear positioning algorithm; methods of reference point indicators; transition point indicator.

Nomenclature

t_i	reference point indicator of $i-th$ point to another point.
$t_{\sum ij \ldots n}$	$n-$reference point indicator
t_a	transition point indicator
$t_{a,\ldots,n}$	$n-$transition point indicator
T_{ab}	the difference of transition point indicators t_a and t_b
$K_{1,1}$	element in the inverse matrix K which consists of partial coordinate differences of the reference points.
Δt_{12_b}	difference of two reference point indicators t_1 and t_2 with regard to the transition point b
X_{ab}	partial coordinates of transition points a and b

1. Introduction

1.1. Reference point indicator definitions

Fixed point indicator definition was proposed by Hausbrandt [1] and was used for linear intersections, and this was published in cracovian form invented by Banachiewicz [2]. On the basis of the fixed point indicator, a definition of reference point indicator was proposed by Oszczak [3, 4, 6]. In two-dimensional space, the reference point indicator t_i of a point $i(x_i, y_i)$ can be defined as sum of the squared reference point coordinates, reduced by the squared distance d_i from the reference point to the determined point. Reference point indicator definition was extended by Oszczak [4] for $n-$reference point indicator definitions which were used for the development of two new linear [4] and iterative [5]

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positioning algorithms. In this publication it has been shown that this abovementioned new linear positioning algorithm can be used in geodetic network solution. According to Oszczak, the n – reference point indicator $t_{\sum ij\ldots n}$ can be defined as the sum of the reference point indicators with regard to other point, or even to many points or distances:

$$t_{\sum ij\ldots n} = t_i + t_j + \ldots + t_n; \tag{1}$$

$$i = (1,2,...,n)$$

$$j = (0,1,2,...,n),$$

where:

t_i – reference point indicator of a point $i(x_i, y_i, z_i)$,

t_j – reference point indicator of a point $j(x_j, y_j, z_j)$

t_n – reference point indicator of point $n(x_n, y_n, z_n)$

The main idea for n – reference point indicators definitions is to associate quadratic distance values with the quadratic reference point coordinates [4]. On the basis of n – reference point indicator definitions, a system of equations can be created and position of sought point can be determined. In the following chapter it has been shown that n – reference point indicator definitions can be used for network point position computation. On the base of the n – reference point indicator definitions, a transition point indicator and n – transition point indicator definitions are introduced.

2. Definition of the transition point indicator and *n* – transition point indicator for the sought point coordinates determination in geodetic network solution

The transition point indicator definition is an extension of the reference point indicators and n – reference point definitions [4] and it is a new concept developed by the author. For the transition point indicators the main idea is to associate quadratic distance values with the quadratic reference point coordinates. If the coordinates of the transition points are not known, the coordinates of the sought point $Q(x_Q, y_Q)$ can be computed on the basis of known distances from these transition points to the point to be determined. It is assumed that the coordinates of both the reference points and distances from reference points to the transition points are known.

For three reference points $1(x_1, y_1)$, $2(x_2, y_2)$, $3(x_3, y_3)$ which coordinates are known, the transition point indicator t_a of the transition point $a(x_a, y_a)$ with regard to the determined point $Q(x_Q, y_Q)$ can be expressed as follows:

$$t_a = \left(\frac{1}{2}\Delta t_{12_a} * K_{1,1} + \frac{1}{2}\Delta t_{13_a} * K_{2,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_a} * K_{1,2} + \frac{1}{2}\Delta t_{13_a} * K_{2,2}\right)^2 - \mathrm{d}_{\mathrm{aQ}}^2, \tag{2}$$

where:

Δt_{12_a} – difference of two reference point indicators t_{2a} and t_{1a} with regard to the transition point $a(x_a, y_a)$

Δt_{13_a} – difference of two reference point indicators t_{3a} and t_{1a} with regard to the transition point $a(x_a, y_a)$

$$\Delta t_{12_a} = t_{2a} - t_{1a}\ ; \Delta t_{13_a} = t_{3a} - t_{1a}$$

$K_{1,1}$, $K_{2,1}$, $K_{1,2}$, $K_{2,2}$ – elements in the inverse matrix K which consists of partial coordinate differences of the reference points.

d_{aQ} – distance from the transition point $a(x_a, y_a)$ to the sought point $Q(x_Q, y_Q)$.

and:

$$\boldsymbol{K} = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} \\ \Delta x_{13} & \Delta y_{13} \end{bmatrix}^{-1}$$

According to the n – reference point indicator definitions the transition n – point indicator t_{an} $(for\, n = 1,...,\infty)$ can be expressed as follows:

$$t_{a,\ldots,n} = t_a + \ldots + t_n, \tag{3}$$

where:

t_n – a transition point indicator of point $n(x_n, y_n)$ with regard to the sought point.

3. New algorithm for determination of the point coordinates in geodetic network solution in two-dimensional space by application of methods of reference and transition point indicators

The transition and reference point indicators can be used for determination of point $Q(x_Q, y_Q)$ in geodetic network solution (Fig. 1).

The coordinates of three reference points are assumed to be:

$$1(x_1, y_1), 2(x_2, y_2), 3(x_3, y_3)$$

There are known distances d_{aQ}, d_{bQ}, d_{cQ} from the transition points $a(x_a, y_a)$, $b(x_b, y_b), c(x_c, y_c)$ to point $Q(x_Q, y_Q)$ which coordinates are to be determined. The distances $d_{1a}, d_{1b}, d_{1c}, d_{2a}, d_{2b}, d_{2c}, d_{3a}, d_{3b}, d_{3c}$ from three reference points $1(x_1, y_1), 2(x_2, y_2), 3(x_3, y_3)$ to the transition points $a(x_a, y_a)$, $b(x_b, y_b)$, $c(x_c, y_c)$ are also known, respectively. However, the coordinates of the transition points $a(x_a, y_a)$, $b(x_b, y_b), c(x_c, y_c)$ are not known.

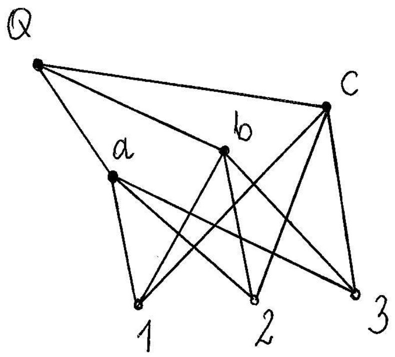

Fig. 1. An example of geodetic linear network solution for sought point $Q(x_Q, y_Q)$ with reference and transition points

$1,2,3-$ the reference points with known coordinates,

$a,b,c-$ the transition points with unknown coordinates

$Q-$ point to be determined.

The coordinates of point $Q(x_Q, y_Q)$ can be computed from the following formula [3]:

$$\begin{bmatrix} x_Q & y_Q \end{bmatrix} = \frac{1}{2}\begin{bmatrix} T_{ab} & T_{ac} \end{bmatrix} \begin{bmatrix} X_{ab} & Y_{ab} \\ X_{ac} & Y_{ac} \end{bmatrix}^{-1}, \tag{4}$$

where:

$$T_{ab} = \left(\frac{1}{2}\Delta t_{12_b} * K_{1,1} + \frac{1}{2}\Delta t_{13_b} * K_{2,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_b} * K_{1,2} + \frac{1}{2}\Delta t_{13_b} * K_{2,2}\right)^2 - \left(\frac{1}{2}\Delta t_{12_a} * K_{1,1} + \frac{1}{2}\Delta t_{13_a} * K_{2,1}\right)^2 + \\ -\left(\frac{1}{2}\Delta t_{12_a} * K_{1,2} + \frac{1}{2}\Delta t_{13_a} * K_{2,2}\right)^2 + d_{aQ}^2 - d_{bQ}^2;$$

$$T_{ac} = \left(\frac{1}{2}\Delta t_{12_c} * K_{1,1} + \frac{1}{2}\Delta t_{13_c} * K_{2,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_c} * K_{1,2} + \frac{1}{2}\Delta t_{13_c} * K_{2,2}\right)^2 - \left(\frac{1}{2}\Delta t_{12_a} * K_{1,1} + \frac{1}{2}\Delta t_{13_a} * K_{2,1}\right)^2 + \\ -\left(\frac{1}{2}\Delta t_{12_a} * K_{1,2} + \frac{1}{2}\Delta t_{13_a} * K_{2,2}\right)^2 + d_{aQ}^2 - d_{cQ}^2; \tag{5}$$

and:

$\Delta t_{12_b} -$ difference of two reference point indicators t_2 and t_1 with regard to the transition point $b(x_b, y_b)$

$\Delta t_{13_b} -$ difference of two reference point indicators t_3 and t_1 with regard to the transition point $b(x_b, y_b)$

$\Delta t_{12_a} -$ difference of two reference point indicators t_2 and t_1 with regard to the transition point $a(x_a, y_a)$

$\Delta t_{13_a} -$ difference of two reference point indicators t_3 and t_1 with regard to the transition point $a(x_a, y_a)$

$\Delta t_{12_c} -$ difference of two reference point indicators t_2 and t_1 with regard to the transition point $c(x_c, y_c)$

$\Delta t_{13_c} -$ difference of two reference point indicators t_3 and t_1 with regard to the transition point $c(x_c, y_c)$

and:

$$X_{ab} = X_b - X_a \;; X_{ac} = X_c - X_a$$

$$Y_{ab} = Y_b - Y_a \;; Y_{ac} = Y_c - Y_a$$

$$\begin{bmatrix} X_b \\ X_a \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \Delta t_{12_b} & \Delta t_{13_b} \\ -\Delta t_{12_a} & -\Delta t_{13_a} \end{bmatrix}\begin{bmatrix} K_{1,1} \\ K_{2,1} \end{bmatrix}; \quad \begin{bmatrix} Y_b \\ Y_a \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \Delta t_{12_b} & \Delta t_{13_b} \\ -\Delta t_{12_a} & -\Delta t_{13_a} \end{bmatrix}\begin{bmatrix} K_{1,2} \\ K_{2,2} \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} X_c \\ X_a \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \Delta t_{12_c} & \Delta t_{13_c} \\ -\Delta t_{12_a} & -\Delta t_{13_a} \end{bmatrix}\begin{bmatrix} K_{1,1} \\ K_{2,1} \end{bmatrix}; \quad \begin{bmatrix} Y_c \\ Y_a \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \Delta t_{12_c} & \Delta t_{13_c} \\ -\Delta t_{12_a} & -\Delta t_{13_a} \end{bmatrix}\begin{bmatrix} K_{1,2} \\ K_{2,2} \end{bmatrix}$$

$$\boldsymbol{K} = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} \\ \Delta x_{13} & \Delta y_{13} \end{bmatrix}^{-1}$$

It has been shown here that the position of sought point $Q(x_Q, y_Q)$ can be determined on the basis of the reference and transition point indicators. Numerical example confirms the correct performance of the presented algorithm. Given data as follows:

$$x_1 = 700\,m; y_1 = 800\,m; x_2 = 400\,m; y_2 = 600\,m; x_3 = 800\,m; y_3 = 250\,m;$$

$$d_{1a} = 8364{,}40\,m; d_{1b} = 10316{,}00\,m\,;\; d_{1c} = 24573{,}00\,m;$$

$$d_{2a} = 8760{,}30\,m; d_{2b} = 10718{,}00\,m; d_{2c} = 25011{,}00\,m;$$

$$d_{3a} = 8766{,}40\,m; d_{3b} = 10704{,}00\,m; d_{3c} = 24844{,}00\,m;$$

$$d_{aQ} = 8207{,}50\,m, d_{bQ} = 10175{,}00\,m, d_{cQ} = 24582{,}00\,m$$

Computation steps are as follows:

$T_{ab} = t_b - t_a = 4483200\,m^2$

$T_{ac} = t_c - t_a = 30742000\,m^2$

$X_{ab} = 1388$ m

$Y_{ab} = 1404$ m

$X_{ac} = 14386$ m

$Y_{ac} = 8504$ m

According to the formula (4) the unknowns x_Q, y_Q can be computed:

$$x_Q = 300\,m \;\; y_Q = 1300\,m$$

For the result confirmation, the geometric distances from $Q(x_Q, y_Q)$ to the transition points can be computed.

Proof. The new linear and iterative positioning algorithms were developed and published by Oszczak [4, 5]. They are based on the n −reference point indicator definitions also invented by the author.
According to Oszczak, the transition points also can be expressed by the $n-$ reference point indicators and partial coordinates of reference points as follows:

$$x_b - x_a = -\frac{1}{2}\Delta t_{12_a} * K_{1,1} + \frac{1}{2}\Delta t_{13_b} * K_{2,1} - \frac{1}{2}\Delta t_{13_a} * K_{2,1} + \frac{1}{2}\Delta t_{12_b} * K_{1,1} \tag{7}$$

$$x_c - x_a = -\frac{1}{2}\Delta t_{12_a} * K_{1,1} + \frac{1}{2}\Delta t_{12_c} * K_{1,1} - \frac{1}{2}\Delta t_{13_a} * K_{2,1} + \frac{1}{2}\Delta t_{13_c} * K_{2,1}$$

$$y_b - y_a = -\frac{1}{2}\Delta t_{12_a} * K_{1,2} + \frac{1}{2}\Delta t_{12_b} * K_{1,2} - \frac{1}{2}\Delta t_{13_a} * K_{2,2} + \frac{1}{2}\Delta t_{13_b} * K_{2,2}$$

$$y_c - y_a = -\frac{1}{2}\Delta t_{12_a} * K_{1,2} + \frac{1}{2}\Delta t_{12_c} * K_{1,2} - \frac{1}{2}\Delta t_{13_a} * K_{2,2} + \frac{1}{2}\Delta t_{13_c} * K_{2,2}$$

The abovementioned linear positioning algorithm can be used for geodetic network solution in the following way.

According to the definition of $n-$reference point indicator, the differences T_{ab}, T_{ac} of transition point indicators t_a, t_b, t_c can be expressed as follows (7):

$$T_{ab} = \left(\frac{1}{2}\Delta t_{12_b} * K_{1,1} + \frac{1}{2}\Delta t_{13_b} * K_{2,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_b} * K_{1,2} + \frac{1}{2}\Delta t_{13_b} * K_{2,2}\right)^2 - \left(\frac{1}{2}\Delta t_{12_a} * K_{1,1} + \frac{1}{2}\Delta t_{13_a} * K_{2,1}\right)^2 +$$

$$-\left(\frac{1}{2}\Delta t_{12_a} * K_{1,2} + \frac{1}{2}\Delta t_{13_a} * K_{2,2}\right)^2 + d_{aQ}^2 - d_{bQ}^2;$$

$$T_{ac} = \left(\frac{1}{2}\Delta t_{12_c} * K_{1,1} + \frac{1}{2}\Delta t_{13_c} * K_{2,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_c} * K_{1,2} + \frac{1}{2}\Delta t_{13_c} * K_{2,2}\right)^2 - \left(\frac{1}{2}\Delta t_{12_a} * K_{1,1} + \frac{1}{2}\Delta t_{13_a} * K_{2,1}\right)^2 +$$

$$-\left(\frac{1}{2}\Delta t_{12_a} * K_{1,2} + \frac{1}{2}\Delta t_{13_a} * K_{2,2}\right)^2 + d_{aQ}^2 - d_{cQ}^2 \;;$$

Coordinates of the transition points are not known but according to the Oszczak linear positioning algorithm for sought point computation, in geodetic network solution partial coordinates of transition points are needed. Thus the author's linear positioning solution [4] can be used directly for the geodetic network solution as follows (4):

$$\begin{bmatrix} x_Q & y_Q \end{bmatrix} = \frac{1}{2}\begin{bmatrix} T_{ab} & T_{ac} \end{bmatrix}\begin{bmatrix} X_{ab} & Y_{ab} \\ X_{ac} & Y_{ac} \end{bmatrix}^{-1}$$

In this publication the numerical example was given for 3 reference and 3 transition points. Moreover, the solution can be extended for any number of reference points and any number of transition points. However, the reference points, nor the transition points cannot be located on a straight line.

4. Conclusion

A new algorithm for determining of the point coordinates in two-dimensional space in geodetic network solution is presented. The definition of the transition point indicator with regard to the determined point has been introduced. Transition point indicator definition is based on the reference point indicator definitions developed by the author. With the use of definitions of these point indicators it is possible to determine the sought point in geodetic network solution.

The direct solution for determining the position of unknown point in geodetic network is derived without application of the least squares method. In the proposed solution there is no need to know the initial approximate location of the determined point. In the presented algorithm there is also no need to know the values of coordinates of transition points. The basic principles of the methods for solving the positioning problem in geodetic networks, the formulas and their derivation has been presented. The numerical example with simulated data and a proof confirm the correct performance of the proposed algorithm.

The presented method should be tested with real measurements in many domains of positioning and navigation as well. In adverse observation conditions [7, 8] the proposed solution also should be further investigated.

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