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Analysis of vertical deflection periodic variations

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Abstract

Most of the geodetic measurements are related with the direction of gravity or vertical. Angular, height difference measurements are performed in horizontal coordinate system. Geodetic measurement data are reduced to a uniform coordinate system prior to processing, taking into account gravity field nonhomogenity. However, increasing accuracy of geodetic measurements requires to assess deviations of the vertical caused by the celestial body effect. The aim of this study is to analyze periodic changes to the direction of the vertical, due to effect of the celestial body, and to assess the impact of the closest celestial bodies which have greater impact. Celestial body effect in the vertical deflection components in meridian and prime vertical was examined. Zonal and nonzonal wave's effect and their amplitude was investigated and evaluated. Results of this work can be used in precise angular measurements and height difference determination, and in the determination of the deflection of the vertical using geodetic astronomy techniques to clarify the shape of the geoid. Results also can be used to assess effect of zonal waves of deflection of the vertical.

Keywords: Deflection of the vertical; tide generating potential; tidal waves.

1. Introduction

Geodetic measurements are performed in the Earth's gravity field which tend to vary within the time running. The angular measurements, determinations of height differences and geodetic astronomy observations are related with the direction of gravity or vertical. Direction of gravity at any point on Earth is perpendicular to the gravity field equipotential surfaces. One of these surfaces is chosen as the geoid, from which the orthometric heights are calculated. The direction of the vertical is determined in respect with the normal to the equipotential ellipsoid i.e. astronomical geodetic deflection of vertical. Also it could be determined with respect to the direction of the normal gravity i.e. the gravimetric deflection of vertical.

The direction of the vertical also can be defined in respect to Earth's equator and the Prime meridian by the astronomical coordinates of the points. Gravity and the direction of vertical continuously changes for a variety of reasons [1, 2]. It is affected by the shift of Earth's masses, also by changes in the atmosphere and hydrosphere. Earths angular rotation speed changes and the movement of the Earth's poles also changes parameters of the gravity field. Significant periodical changes of vertical or deflections of vertical are caused by attraction of closer located and heavy weighted celestial bodies. Mainly it's the Moon and the Sun pull [3]. Increasing accuracy of the geodetic and astronomical measurements [4, 5, 6, 7] requires a deeper analysis of the celestial bodies effects on the vertical deviation and more precise assessment of these effects. This paper presents the research results of the vertical deviation periodic change due to effects of celestial bodies. The influence of the Moon and the Sun to the deflection of the vertical is estimated.

2. Deflections of the vertical triggered by the celestial body

Influence of the celestial body on the Earth gravity field can be expressed using tide generating potential. Using the celestial equatorial coordinate system, tide generating potential of the non-deformable (rigid) Earth can be expressed by formula [8]:

$$V_T = \sum_{n=2}^{\infty} V_{Tn} \,, \tag{1}$$

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where

$$V_{Tn} = \frac{Gm}{r}\left(\frac{R}{r}\right)^n \left[P_n(\sin\Phi) P_n(\sin\delta) + 2\sum_{k=1}^{n} \frac{(n-k)!}{(n+k)!} P_n^k(\sin\Phi) P_n^k(\sin\delta)\cos kt \right], \quad (2)$$

where G – is a gravitational constant; m – is the mass of the celestial body; r – is the geocentric distance to the celestial body; R – is the geocentric distance to the point of the Earth's surface; Φ – is the geocentric latitude of the point; δ – is the declination of the celestial body; t – is the hour angle of the celestial body in respect with the local meridian; $P_n(\sin\Phi)$ and $P_n(\sin\delta)$ – is the Legendre polynomials; $P_n^k(\sin\Phi)$ and $P_n^k(\sin\delta)$ – is the appended Legendre functions.

Taking into account current accuracy of geodetic measurements, tide generating potential series may be limited to the first three members [8]:

$$V_{T2} = \frac{GmR^2}{4r^3}\left[\left(3\sin^2\delta - 1\right)\left(3\sin^2\Phi - 1\right) + 3\sin 2\delta \sin 2\Phi \cos t + 3\cos^2\delta\cos^2\Phi\cos 2t\right], \quad (3)$$

$$V_{T3} = \frac{GmR^3}{4r^4}\left[\left(5\sin^3\delta - 3\sin\delta\right)\left(5\sin^3\Phi - 3\sin\Phi\right) + \frac{3}{2}\cos\delta\left(5\sin^2\delta - 1\right)\cos\Phi\left(5\sin^2\Phi - 1\right)\cos t + \right.$$
$$\left. 15\cos^2\delta\sin\delta\cos^2\Phi\sin\Phi\cos 2t + \frac{5}{2}\cos^3\delta\cos^3\Phi\cos 3t\right] \quad (4)$$

$$V_{T4} = \frac{GmR^4}{r^5}\left[\frac{1}{64}\left(35\sin^4\delta - 30\sin^2\delta + 3\right)\left(35\sin^4\Phi - 30\sin^2\Phi + 3\right) + \right.$$
$$\cos\Phi\left(7\sin^3\Phi - 3\sin\Phi\right)\cos t + \frac{5}{16}\cos^2\delta(7\sin^2\delta - 1)\cos^2\Phi(7\sin^2\Phi - 1)\cos 2t +$$
$$\left. \frac{5}{8}\cos\delta\left(7\sin^3\delta - 3\sin\delta\right) \ \frac{35}{8}\cos^3\delta\sin\delta\cos^3\Phi\sin\Phi\cos 3t + \frac{35}{64}\cos^4\delta\cos^4\Phi\cos 4t\right]. \quad (5)$$

First members of the tide potential formulas can be used to evaluate the impact of celestial bodies to the direction of gravity.

Vertical deflection can be decomposed into two components: ξ in the meridian plane, and η in the prime vertical plane. The components of vertical deflection can be expressed using tide generating potential by following equations:

$$\xi^T = -\frac{1}{gR}\frac{\partial V_T}{\partial\Phi}, \quad (6)$$

$$\eta^T = -\frac{1}{gR\cos\Phi}\frac{\partial V_T}{\partial\Lambda}, \quad (7)$$

where g – is gravity acceleration; Λ – geocentric longitude of the point. For individual tide potential members can be written.

$$\xi_n^T = -\frac{1}{gR}\frac{\partial V_{Tn}}{\partial\Phi}, \quad \eta_n^T = -\frac{1}{gR\cos\Phi}\frac{\partial V_{Tn}}{\partial\Lambda}. \quad (8)$$

By differentiating Equations (3)–(5) by latitude we can express vertical deflection in meridian plane by formula:

$$\xi_2^T = -\frac{3GmR}{4gr^3}\left[\left(3\sin^2\delta - 1\right)\sin 2\Phi + 2\sin 2\delta\cos 2\Phi\cos t - \cos^2\delta\sin 2\Phi\cos 2t\right], \quad (9)$$

$$\xi_3^T = -\frac{GmR^2}{4gr^4}\left[\left(5\sin^3\delta - 3\sin\delta\right) 3\cos\Phi\left(5\sin^2\Phi - 1\right) + \frac{3}{2}\cos\delta\left(5\sin^2\delta - 1\right)\sin\Phi\left(15\cos^2\Phi - 4\right)\cos t + \right.$$
$$\left. 15\cos^2\delta\sin\delta\cos\Phi\left(1 - 3\sin^2\Phi\right)\cos 2t - \frac{15}{2}\cos^3\delta\cos^2\Phi\sin\Phi\cos 3t\right], \quad (10)$$

$$\xi_4^T = -\frac{GmR^3}{gr^5}\left[\frac{5}{32}\left(35\sin^4\delta - 30\sin^2\delta + 3\right)\left(7\sin^2\Phi - 3\right)\sin 2\Phi + \frac{5}{8}\cos\delta\left(7\sin^3\delta - 3\sin\delta\right)\right.$$

$$\left[\cos 2\Phi\left(7\sin^2\Phi-3\right)+\frac{7}{2}\sin^2 2\Phi\right]\cos t\ \frac{5}{8}\cos^2\delta\left(7\sin^2\delta-1\right)\sin 2\Phi\left(4\cos^2\Phi-3\sin^2\Phi\right)\cos 2t+$$

$$\frac{35}{8}\cos^3\delta\sin\delta\cos^2\Phi\left(\cos^2\Phi-3\sin^2\Phi\right)\cos 3t-\frac{35}{16}\cos^4\delta\cos^3\Phi\sin\Phi\cos 4t\Bigg]. \quad (11)$$

Some members in the above Equations (9)–(11), are dependant only from declination of celestial body and local latitude. This is zonal tidal waves in meridian component of the vertical deflection. The remaining members depend from the hour angle of the celestial body and represent the tesseral and sectorial tidal waves. The period of these waves changing from day till n parts of day.

Similarly, we will get components of the vertical deflection members at the first vertical:

$$\eta_2^T=\frac{3GmR}{2gr^3}\left[\sin 2\delta\sin\Phi\sin t+\cos^2\delta\cos\Phi\sin 2t\right], \quad (12)$$

$$\eta_3^T=\frac{3GmR^2}{4gr^4}\left[\frac{1}{2}\cos\delta\left(5\sin^2\delta-1\right)\left(5\sin^2\Phi-1\right)\sin t+10\cos^2\delta\sin\delta\cos\Phi\sin\Phi\sin 2t+\frac{5}{2}\cos^3\delta\cos^2\Phi\sin 3t\right], \quad (13)$$

$$\eta_4^T=\frac{5GmR^3}{gr^5}\left[\frac{1}{8}\cos\delta\left(7\sin^3\delta-3\sin\delta\right)\left(7\sin^3\Phi-3\sin\Phi\right)\sin t+\frac{1}{8}\cos^2\delta\left(7\sin^2\delta-1\right)\cos\Phi\left(7\sin^2\Phi-1\right)\sin 2t+\right.$$

$$\left.\frac{21}{8}\cos^3\delta\sin\delta\cos^2\Phi\sin\Phi\sin 3t+\frac{7}{16}\cos^4\delta\cos^3\Phi\sin 4t\right]. \quad (14)$$

From the Equations (12)–(14) we see, that vertical deviation in prime vertical plane does not have any zonal waves. This component of vertical deviation has only tesseral and sectoral waves. Periods of waves depends on the n and hour angle.

During assessment of the impact of the celestial body on the deflection of the vertical in respect with the surface of the Earth, the Earths elasticity coefficient γ_n is used:

$$\xi^\gamma=\sum_{n=2}^{\infty}\gamma_n\xi_n^T=\sum_{n=2}^{\infty}\xi_n^\gamma, \quad (15)$$

$$\eta^\gamma=\sum_{n=2}^{\infty}\gamma_n\eta_n^T=\sum_{n=2}^{\infty}\eta_n^\gamma, \quad (16)$$

where $\xi_n^\gamma=\gamma_n\xi_n^T$, $\eta_n^\gamma=\gamma_n\eta_n^T$.

If we want to assess the impact of the celestial body on the deflection of the vertical in respect with the rotation axis of the Earth, the Earths elasticity coefficient β_n is used:

$$\xi^\beta=\sum_{n=2}^{\infty}\beta_n\xi_n^T=\sum_{n=2}^{\infty}\xi_n^\beta, \quad (17)$$

$$\eta^\beta=\sum_{n=2}^{\infty}\beta_n\eta_n^T=\sum_{n=2}^{\infty}\eta_n^\beta \quad (18)$$

where $\xi_n^\beta=\beta_n\xi_n^T$, $\eta_n^\beta=\beta_n\eta_n^T$.

Other Earth elasticity coefficients are expressed using the following equations:

$$\gamma_n=1+k_n-h_n, \quad (19)$$

$$\beta_n=1+k_n-l_n, \quad (20)$$

where k_n, h_n ir l_n – Love numbers (Table 1).

Table 1. Love numbers [9]

n	k_n	h_n	l_n	γ_n	β_n
2	0.2993	0.6053	0.0841	0.6946	1.2152
3	0.0925	0.2890	0.0147	0.8036	1.0778
4	0.0417	0.1758	0.0101	0.8654	1.0316

3. Zonal waves of the vertical deviation

All fi We will examine the impact of tidal potential zonal waves on vertical deviation component in meridian. This impact can be expressed by the formula:

$$\xi_z^T = \frac{3GmR}{4gr^3}\left(1-3\sin^2\delta\right)\sin 2\Phi + \frac{3GmR^2}{4gr^4}\left(5\sin^3\delta - 3\sin\delta\right)\cos\Phi\left(1-5\sin^2\Phi\right)+$$

$$\frac{5GmR^3}{32gr^5}\left(35\sin^4\delta - 30\sin^2\delta + 3\right)\left(3-7\sin^2\Phi\right)\sin 2\Phi \,. \tag{21}$$

We will evaluate functions depending on δ by taking its mean integral values. For that purpose we will use relation of declination with celestial body longitude in orbit λ and deviation of the orbit on the equator ε. The relation is described by equation: sin δ = sin ε sin λ. By adopting that longitude of celestial body vary from 0 to 2π, we will get mean integral values of*δ* function:

$$f\left(\delta\right)_{2v} = \frac{1}{2\pi}\int_0^{2\pi} 3\sin^2\delta\cdot d\lambda = \frac{3}{2}\sin^2\varepsilon \,; \tag{22}$$

$$f\left(\delta\right)_{3v} = \frac{1}{2\pi}\int_0^{2\pi} \left(5\sin^3\delta - 3\sin\delta\right)d\lambda = 0 \,; \tag{23}$$

$$f\left(\delta\right)_{4v} = \frac{1}{2\pi}\int_0^{2\pi} \left(35\sin^4\delta - 30\sin^2\delta\right)d\lambda = \frac{105}{8}\sin^4\varepsilon - 15\sin\varepsilon \,. \tag{24}$$

Using received mean values we will get following zonal waves of vertical deviation:

$$\xi_{zv}^T = \frac{3GmR}{4gr^3}\left(1-\frac{3}{2}\sin^2\varepsilon\right)\sin 2\Phi + \frac{5GmR^3}{32gr^5}\left(\frac{105}{8}\sin^4\varepsilon - 15\sin\varepsilon + 3\right)\left(3-7\sin^2\Phi\right)\sin 2\Phi \,. \tag{25}$$

In the above formulas we can exclude the zonal wave depending only on the latitude.

$$\xi_{z\Phi}^T = \frac{3GmR}{4gr^3}\sin 2\Phi + \frac{15GmR^3}{32gr^5}\left(3-7\sin^2\Phi\right)\sin 2\Phi \,. \tag{26}$$

This part of the zonal wave of the vertical deviation in meridian plane is not changing over the time and expresses permanent part of vertical deflection influenced by the celestial body.

4. Evaluation of the zonal wave in vertical deviation

Using the (26) and (19) equations we will get the formula for a permanent vertical deviation in respect with the surface of the Earth:

$$\xi_{z\Phi}^T = \frac{\gamma_2 3GmR}{4gr^3}\sin 2\Phi + \frac{\gamma_4 15GmR^3}{32gr^5}\left(3-7\sin^2\Phi\right)\sin 2\Phi \,. \tag{27}$$

Permanent zonal wave effect on the vertical deviation, depending on the latitude, when using the maximum distance from the celestial body (406 700 km for Moon and 152 097 701 km for Sun) is presented in the figure 1. (selenocentric constant of gravity – Gm_M = 4902,799 km^{3}s^{-2} heliocentric constant of gravity – Gm_S = 132 712 440 018 km^{3}s^{-2} , ellipsoid GRS80). It should be noted that the vertical deviation doesn't have a permanent part at the first vertical. Therefore only permanent part of deviation at the meridian is analyzed.

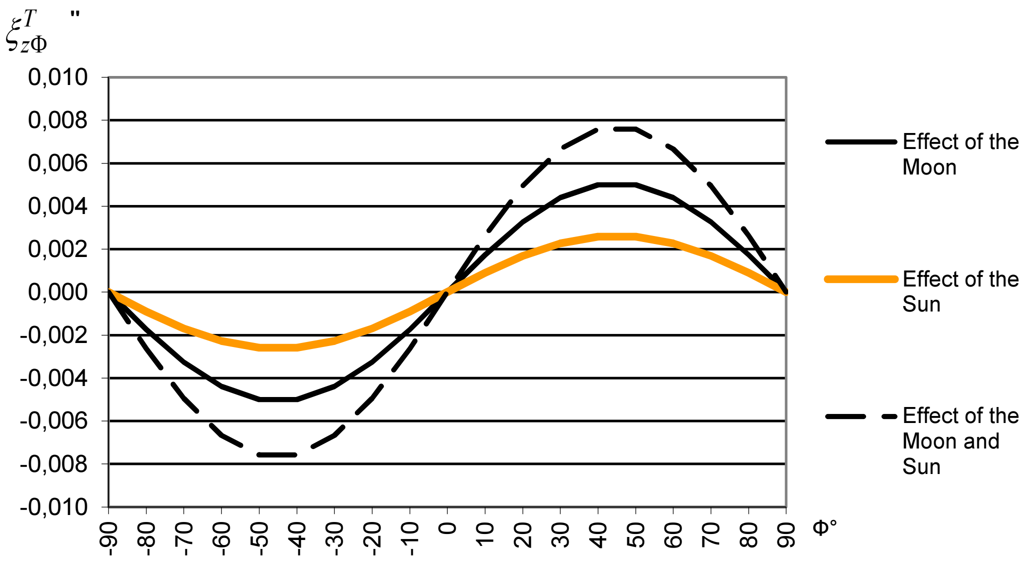

Fig. 1. Permanent change of deflection of vertical in meridian (n = 2, 4)

The research shows that a permanent vertical deviation in the meridian, changes from 0"at the poles and equator; to ±0.0051" when the latitude is ±45°, due to impact of the Moon. The impact of the Sun is smaller and these deviations could vary from 0 " at the poles and equator to ± 0.0026" when the latitude is ±45°. The total effect of the both celestial bodies is causing permanent vertical deviation which depends only from latitude. The ranges of this deviation could vary from 0" at the poles and equator to ±0.0077" when the latitude is ±45°. From the southern to the northern part of Lithuania the permanent vertical deviation caused by both celestial bodies is changing from 0.0073" till 0.0071".

Using the function of the average integral celestial body declinations, by the (25) and (19) formulas we can write an equation for vertical deviation at the meridian.

$$\xi^T_{zv} = \frac{\gamma_2 3GmR}{4gr^3}\left(1-\frac{3}{2}\sin^2\varepsilon\right)\sin 2\Phi + \frac{\gamma_4 5GmR^3}{32gr^5}\left(\frac{105}{8}\sin^4\varepsilon - 15\sin\varepsilon + 3\right)\left(3-7\sin^2\Phi\right)\sin 2\Phi \tag{28}$$

The zonal wave effect on the vertical deviation at the maximum and minimum distance from the celestial body (the minimum distance to the Moon 356 400 km and to the Sun – 147 098 074 km) is presented in Figure 2.

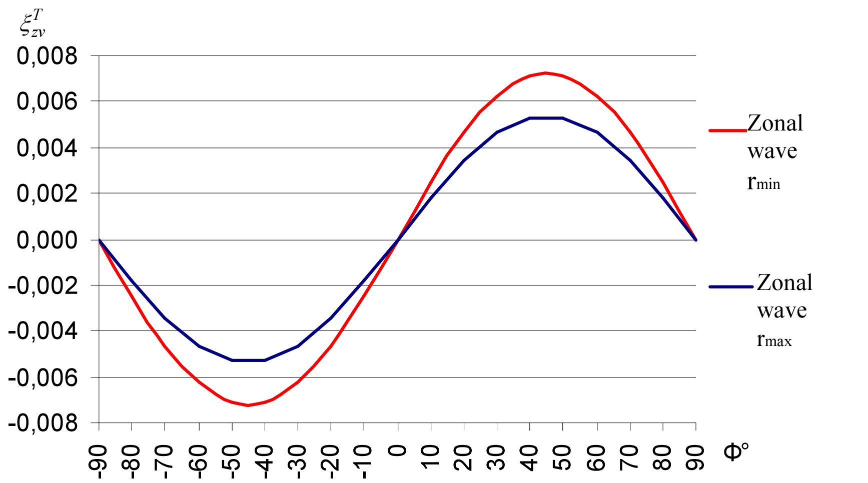

Fig. 2. Deflection of vertical caused by zonal function for elastic Earth (n=2, n=4)

Results of research shows, that vertical deviation in the meridian is equal to zero at the poles and the equator. When the geocentric distance to celestial bodies is biggest, the vertical deviate up to ±0.0054" at the latitude of ±45°. The vertical deviation in the meridian, when the geocentric distance to the Moon and to the Sun is smallest, is equal zero at the poles and equator. Deviation of vertical reaches ±0.0072" at the ±45° latitude. Changes of geocentric distance to celestial bodies can change vertical deviation range up to 0.0016" because of the Moon effect and 0.0002" because of the Sun effect.

5. Conclusions

1. Based on the theory of tide potential, the effects of the celestial body to the vertical deviation in the meridian and prime vertical were investigated. The finite equations for each member of the tidal potential were received. The zonal waves of the vertical deviation, depending from the latitude and declination and geocentric distance of celestial body, were derived separately.
2. The effects of the zonal waves were estimated taking into account the elasticity of the Earth, using the derived formulas. The direction of the vertical is changing in range of $\pm 0,0077''$. Permanent vertical deviation, depending on influence of both celestial bodies, is changing from 0,0071" till 0,0073" in territory of Lithuania.
3. Received results can be used in the precise angular and height difference measurements. Also it can be used in determination of the vertical deviation by geodetic astronomy methods as well as in selecting equipotential surface of the gravity field to define the shape of geoid surface

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