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Section: Technologies of Geodesy and Cadastre

About accuracy of analytical determination of areas for cadastre and other purposes

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Abstract

At present, when digital maps are produced, areas are either calculated with the use of analytical-and-digital methods or they are directly acquired from databases. Besides the area of objects is a key attribute stored in cadastral registers and for other geodetic purposes. In the process of development and modernization or updating lands or buildings register database the knowledge of the certainty of analytical determination of areas is required. The knowledge of accurately determined areas is also indispensable for other activities, as e.g. in the case of geodetic maintenance of investments or in the process of control of direct subsidies for agriculture, which are performed within the Integrated Administration and Control System (IACS).
The paper considers the case of the relative accuracy of calculation the area of the geometric figure (error-free assumption for the geodetic control network points) basing on mean square errors of X, Y coordinates of its vertices.
A formula, which defines the accuracy of the calculation the area of the polygon, was determined from well known Gauss' equation on double area calculation. In analyses was stated permissible uncertainty of calculation of an area from coordinates of vertices' points for different methods of producing cadastral and other digital map data.

Keywords: digital map data; areas of cadastral objects; error of areas determination.

Nomenclature

m_A	error of analytical calculation of area of (m^2)
A	area of geometric figures (such as cadastral parcels) of (m^2 or ha)
m_P	position error of the vertex of (m)
d_i	diagonals of a polygon, there is the shortest diagonal which is vis-à-vis of vertex point number *i*-th of (m)
n	number of sides of polygon (of a geometric figure)
k	coefficient of elongation of the geometric figure
O	circumference length of the polygon (of a geometric figure) of (m)
s	maximum width of the polygon (of a geometric figure) of (m)
dA_{max}	maximum values of the permissible differences of double determination of area from coordinates of (m^2)

1. Introduction

The typical tasks of large-scale maps are determination of lengths of linear objects and facilitation of calculation of areas. At present, when digital maps are produced, areas are either calculated with the use of analytical-and-digital methods [1] or they are directly acquired from databases. Besides the area of objects is a key attribute stored in lands and buildings registers; they often contain information required for correct implementation of land management works as well as for make real of investments.

So far, the values of applied permissible differences of double calculation of areas are determined basing on formulae that express it in general as a function of size of the area and, sometimes, the elongation of a geometric figure investigated. Meanwhile, that – besides the size and shape – the accuracy of analytical calculation of area of a geometric figure depends on errors of position determination of its vertices [2, 3, 4].

In the time of analyzing the accuracy of calculation of area basing on coordinates, their accuracy should be considered [5]. With respect to speed and economic effectiveness of acquisition of field data for calculation of areas, the use of modern

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satellite positioning techniques seems to be very promising. As early as in the beginning of civil, measuring applications of the GPS system, advantages of area determination using GPS surveying technique were noticed [6, 7, 8]. Besides, in the context of wider economic applications, in land information systems and the modern cadastre and related tasks (as for the Land Parcel Identification System (LPIS) [9]), besides direct measurements with the use of an total station [10] or the RTK GPS technique [11], digital photogrammetry and remote sensing techniques are also applied [12]. Modern surveying techniques are commonly applied as mutually amending techniques [13], and they are supported by GIS applications in specialized information systems [14].

Present techniques of surveying allows for the determination of topographic features with high accuracy. However, it should be noted, that in the case of estimation of the accuracy using the absolute error measure and considering errors of the geodetic control network points, the accuracy of position of geodetic control network and its correct geometric construction is of key importance [15]. Estimation of the accuracy of analytical calculation of areas using the absolute error measure should also consider the accuracy of the geodetic control [16, 17, 18]. When the relative accuracy is considered, the error-free assumption is made for the geodetic control network points. For field measurements, such cases may include measurements performed in the local coordinate frame of the measuring instrument or double measurements performed with the use of identical geodetic control network points.

The paper considers the case of the relative accuracy of calculation the area of the geometric figure (error-free assumption for the geodetic control network points) basing on mean square errors of X, Y coordinates of its vertices. This paper also presents the equation of determination of permissible differences of double determination of the area for cadastral and other objects, using the coordinates of vertices of border lines, calculated on the basis of two independent measurements of the same accuracy.

2. Assessment of accuracy of analytical calculation of areas

In the literature [2, 3] it is known the equation for calculation of the error of an area (m_A) of a polygon of n sides, depending on coordinates of its vertices:

$$m_A = \pm\sqrt{1/4 \cdot \sum_{i=1}^{n} m_i^2 \cdot (2 \cdot m_{i+1}^2 + d_i^2)} \tag{1}$$

where m_i is the error of determination of coordinates of the i^{th} vertex, and d_i are diagonals of a polygon (Fig. 1), which connect $i - 1$ and $i + 1$ vertices ($i = 1, 2, \ldots, n$, providing that vertices numbered 0 and $n + 1$ determine the vertices numbered n and 1, respectively).

The exact Eqn (1) may be simplified after consideration, that $2 \cdot m_{i+1}^2 << d_i^2$ – and under the assumption of equal accuracy of determination of X, Y coordinates of vertices (i.e. $m_1 = m_2 = \ldots = m_n = m$) and of lack of correlation between coordinates of the same point [2]. It is also assumed that the accuracy of vertices is not affected by errors of the control network points.

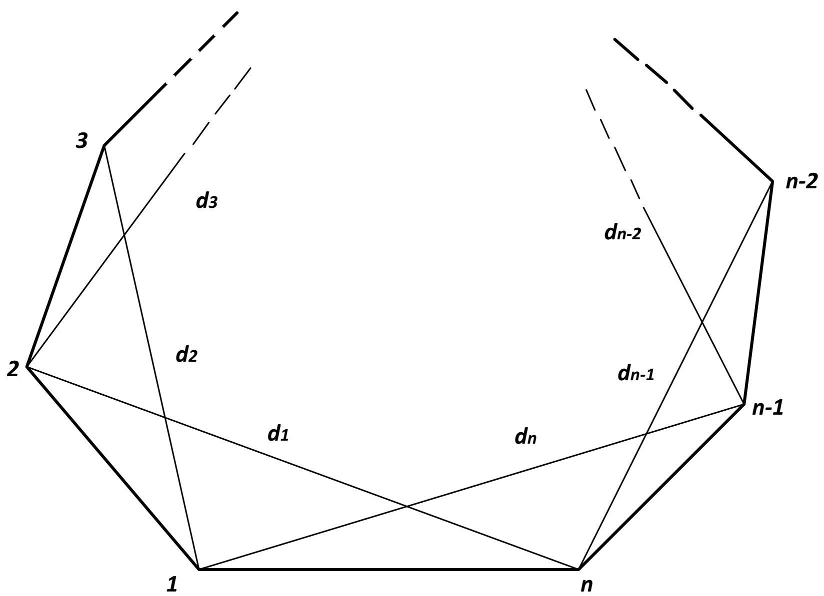

Fig. 1. Location of diagonals opposite to successive vertices of the polygon (source: own elaboration on the basis of [3])

Accordance with [3], the simplified equation ensures in practice the required accuracy of determination of an error of analytical area calculation and the identical accuracy of position of all vertices (m_P) was assumed:

$$m_A = m_P \cdot \sqrt{\tfrac{1}{8} \sum_{i=1}^{n} d_i^2} \tag{2}$$

where m_P is the position error of the vertex ($m_P = \sqrt{m_x^2 + m_y^2} = \sqrt{2 \cdot m^2}$, and thus $m = m_P / \sqrt{2}$).

The Eqn (2) may be fixed (see e.g. [19]) from Gauss' equation $2 \cdot A = \sum_{i=1}^{n} y_i \cdot (x_{i-1} - x_{i+1})$ after consideration its a developed form $2 \cdot A = y_1 \cdot (x_n - x_2) + \ldots + y_n \cdot (x_{n-1} - x_1)$. Under the assumption of lack of correlation between coordinates of the vertices, the equation for an error of analytical area calculation is as follow:

$$m_A = \sqrt{\tfrac{1}{4} \cdot \left\{ m_{y_1}^2 \cdot (x_n - x_2)^2 + \ldots + m_{y_n}^2 \cdot (x_{n-1} - x_1)^2 + m_{x_1}^2 \cdot (y_2 - y_n)^2 + \ldots + m_{x_n}^2 \cdot (y_1 - y_{n-1})^2 \right\}}$$

Under the assumption of the identical accuracy of coordinates of all vertices $m_{x_i} = m_{y_i} = m_i$ (where the position error of the vertex $m_P = \sqrt{m_{x_i}^2 + m_{y_i}^2} = \sqrt{2 \cdot m_i^2}$):

$$m_A = \sqrt{\tfrac{1}{4} \sum_{i=1}^{n} \left\{ (x_{i+1} - x_{i-1})^2 + (y_{i+1} - y_{i-1})^2 \right\} \cdot m_i^2}$$

$$m_A = \sqrt{\tfrac{1}{8} \sum_{i=1}^{n} \left\{ (x_{i+1} - x_{i-1})^2 + (y_{i+1} - y_{i-1})^2 \right\} \cdot m_P^2}$$

$$m_A = m_P \cdot \sqrt{\tfrac{1}{8} \sum_{i=1}^{n} d_{i-1,i+1}^2}$$

where $d_{i-1,i+1}$ is the shortest diagonal which is vis-à-vis of vertex point number *i*-th.

In accordance with result of previous investigations was stated that is the influence of number of vertices of a polygon on the accuracy of analytical determination of its area [4]. That relation presented using the example of a regular polygon, of the size of 1 ha and the number of sides *n* equal to: 4, 5, 6, 7 – under the assumption of the same accuracy of determination of position of its vertices (assuming m_P equal: 0.01 m, 0.03 m, 0.05 m, 0.10 m, 0.30 m, and 0.50 m) is illustrated in Figure 2.

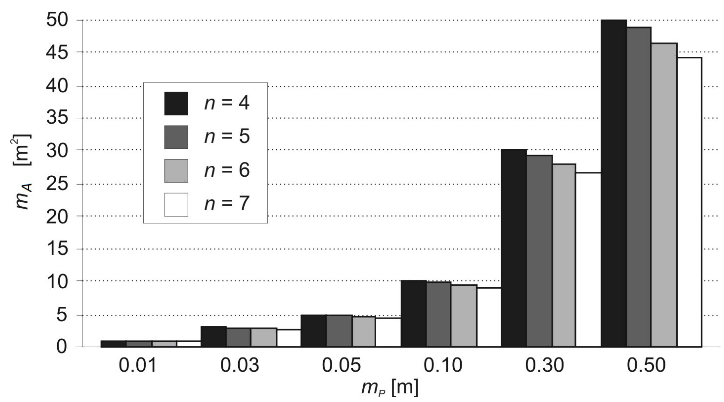

Fig. 2. Influence of the number of sides of a regular polygon on the accuracy of calculation of its area (source: own elaboration in the Microsoft Excel)

Values of the area determination error, presented in Figure 2, were obtained using the equation resulting from transformation of (2) under the assumption of regularity of a geometric figure [2, 19]

$$m_A = m_P \cdot \sqrt{A} \cdot \sqrt{\sin(360°/n)} \tag{3}$$

The Eqn (3) represents the accuracy of the area determination in terms of its size *A*, the position error of vertices m_P and the number *n* of sides (number of vertices) of the regular polygon. In case of surface objects (closed polygons) the number of sides equals to the number of vertices.

In this connection, analysis of the accuracy of area calculation of rectangles using coordinates, may be justified by the fact that the increase of the number of sides (when the area is constant) results in shortening of the length of diagonals d_i of the figure, and, therefore, in decrease of the area determination error, see Eqn (2).

Assuming that the polygon, which area is being calculated is a rectangle of the length a and the width b and considering that the coefficient of elongation $k = a/b$ and the area $A = a \cdot b$ ($a^2 = A \cdot k$ and $b^2 = A/k$) the Eqn (2) may be presented in the following form [19]:

$$m_A = m_P \cdot \sqrt{A} \cdot \sqrt{(1+k^2)/2 \cdot k} \tag{4}$$

In case of surface objects of rectangular shapes, calculation of the elongation coefficient k does not cause any problems, since it is determined by the length to the width ratio. However, in case of irregular shapes of polygons, in order to determine the elongation coefficient k one must know the circumference length O of the polygon and its maximum width s. Then the ratio $(\frac{O}{2} - s)/s$ may represent the elongation coefficient k [20].

3. Accuracy analysis of area determination basing on coordinates

Accuracy of analytical determination of area of geometric figures (such as cadastral parcels) basing on X, Y plane coordinates, depends linearly on the accuracy of determination of position of their vertices (under the assumption of equal accuracy of position of border points). Mean square errors of coordinates, totally expressed by the error of point position.

In the paper accuracy of analytical area determination of a geometric figure of rectangular shape using the Eqn (4) was evaluated. Values of the error of determination of the area examples of the rectangle with elongation coefficient k ranging from 1 to 15, basing on coordinates of vertices, which position have been determined with accuracy required in cadastral measurements in Poland (m_P = 0.10 m, with respect to the closest points of the horizontal geodetic control) are given in Table 1.

Table 1. Accuracy of analytical determination of area A basing on coordinates of vertices, which position has been determined with the accuracy m_P = 0.10 m

Area of a figure A [ha]	Error of analytical determination of area using the Eqn (4) [m^2]									
	Elongation coefficient k of a figure									
	1	1.5	2	2.5	3	4	5	7	10	15
0.01	1.0	1.0	1.1	1.2	1.3	1.5	1.6	1.9	2.2	2.7
0.1	3.2	3.3	3.5	3.8	4.1	4.6	5.1	6.0	7.1	8.7
0.5	7.1	7.4	7.9	8.5	9.1	10.3	11.4	13.4	15.9	19.4
1	10.0	10.4	11.2	12.0	12.9	14.6	16.1	18.9	22.5	27.4
3	17.3	18.0	19.4	20.9	22.4	25.2	27.9	32.7	38.9	47.5
7	26.5	27.5	29.6	31.9	34.2	38.6	42.7	50.0	59.5	72.6
10	31.6	32.9	35.4	38.1	40.8	46.1	51.0	59.8	71.1	86.8
20	44.7	46.5	50.0	53.9	57.7	65.2	72.1	84.5	100.5	122.7
50	70.7	73.6	79.1	85.1	91.3	103.1	114.0	133.6	158.9	194.1
100	100.0	104.1	111.8	120.4	129.1	145.8	161.2	189.0	224.7	274.5
1000	316.2	329.1	353.6	380.8	408.2	461.0	509.9	597.6	710.6	867.9

Values of errors of analytical area determination obtained prove, that in case of a parcel of 1 ha area, uncertainty of determination of its area (basing on coordinates of border points, which position has been determined with the accuracy of 0.10 m), varies between 10 m^2 and 30 m^2 (for discussed values of the k coefficient), depending on its elongation. That result seems to be unsatisfactory, and because the area of a cadastral parcel is calculated basing on coordinates and is specified in hectares with the accuracy of notation of 0.0001, is rather unsettling.

Seems to be reasonable to the relations discussed above should result in determining of certain criteria concerning the precision of notation (determination) of areas. For example, in the Appendix E3 to „General Instructions for Surveys of Canada Lands" Earth Sciences Sector of Natural Resources Canada, the guidelines concerning the precision of notation of parcel areas are following [21]: areas of the parcel up to 0.1 ha are quote to 1 m^2, over 0.1 ha to 1.0 ha are quote to 10 m^2, over 1.0 ha to 10.0 ha are quote to 100 m^2, over 10.0 ha to 100.0 ha are quote to 1000 m^2 and areas over 100 ha are quote to 10000 m^2 (1 ha).

4. Permissible differences of double determination of the area from coordinates

In the process of creation, updating and modernisation of lands and buildings register databases, the knowledge of accuracy of analytical determination of areas is required. Besides, when areas of cadastral objects are calculated basing on results of two measurements of the same accuracy from coordinates of the vertices of border lines – values of differences in calculated areas (resulting from the law of propagation of errors) should be considered with respect to their permissibility.

The method of determination of the permissible values of differences of double determination of areas of cadastral objects basing on points of apex of borders (with equal accuracy of determination of coordinates) is presented in this section.

The area difference $dA = A_1 - A_2$ between the first (A_1) and the second (A_2) area calculation (from the measurements fulfilling the same standard) performed with the accuracy specified by the Eqn (2), is due to the position errors m_P of the vertices. The mean square error of the difference of double determination of the area is obtained as follows

$$m_{dA}^2 = m_{A_1}^2 + m_{A_2}^2 \tag{5}$$

and, with the assumption of the same accuracy of determination of coordinates of vertices, and thus the same accuracy of analytical determination of the area $m_{A_1}^2 = m_{A_2}^2 = m_A^2$, m_{dA} is expressed by the equation

$$m_{dA} = \sqrt{2} \cdot m_A = \sqrt{2} \cdot m_P \cdot \sqrt{1/8 \sum_{i=1}^{n} d_i^2} \tag{6}$$

Then considering that the area has been correctly determined, when the difference between two determinations does not exceed the double value of the mean square error of individual determination of the area (2), the following equation for determination of the maximum, permissible difference of double determination of the area of a geometric figure basing on coordinates of the points of apex of its borders was obtained:

$$dA_{\max} = 2 \cdot m_{dA} = 2 \cdot \sqrt{2} \cdot m_P \cdot \sqrt{1/8 \sum_{i=1}^{n} d_i^2} = \sqrt{8} \cdot m_P \cdot \sqrt{1/8 \sum_{i=1}^{n} d_i^2} = m_P \cdot \sqrt{\sum_{i=1}^{n} d_i^2} \tag{7}$$

Under the assumption that the figure of the calculated area, is a rectangle, the Eqn (7) may have the following form:

$$dA_{\max} = 2 \cdot m_P \cdot \sqrt{A} \cdot \sqrt{(1+k^2)/k} \tag{8}$$

Maximum values of the permissible differences of double determination of areas of rectangular cadastral objects, calculated with the use of Eqn (8) are given in Table 2. The presented values express relative values of differences (calculated without consideration of position errors of points of the horizontal network). However, it should be stressed, that the maximum values of the permissible differences of double determination of areas from coordinates refer to such cases when those calculations are performed basing on coordinates of topographic features determined with the required accuracy $m_P \leq 0.10$ m with relation to the points of the horizontal control network.

Table 2. Maximum values of permissible differences of double determination of areas of rectangles from coordinates of vertices (for $m_P = 0.10$ m)

Area of an object [ha]	Permissible deviation of double calculation of area using the Eqn (8) [m^2]									
	Elongation coefficient of an object									
	1	1.5	2	2.5	3	4	5	7	10	15
0.01	3	3	3	3	4	4	5	5	6	8
0.1	9	9	10	11	12	13	14	17	20	25
0.5	20	21	22	24	26	29	32	38	45	55
1	28	29	32	34	37	41	46	54	64	78
3	49	51	55	59	63	71	79	93	110	135
5	63	66	71	76	82	92	102	120	142	174
7	75	78	84	90	97	109	121	141	168	205
10	89	93	100	108	116	130	144	169	201	246
20	127	132	141	152	163	184	204	239	284	347
50	200	208	224	241	258	292	323	378	449	549
100	283	294	316	341	365	412	456	535	636	776
200	400	416	447	482	516	583	645	756	899	1098
500	633	658	707	762	817	922	1020	1195	1421	1736
1000	894	931	1000	1077	1155	1304	1442	1690	2010	2455
2000	1265	1317	1414	1523	1633	1844	2040	2391	2843	3472
5000	2000	2082	2236	2408	2582	2916	3225	3780	4494	5489
10000	2828	2944	3162	3406	3652	4123	4561	5345	6356	7763

After consideration the rectangular cadastral parcel, for example of the size of 1 ha with different size the elongation coefficient (under the assumption equal accuracy of position determination of its border points m_P = 0.10 m) the values of the permissible difference in the area calculated from coordinates equals from 28 m^2 to 78 m^2 (Table 2).

5. Final remarks and conclusions

Analytical determination of areas of cadastral objects (and other surface objects) basing on plane coordinates of their vertices, acquired from the database of a digital map or by means of modern measuring techniques, meets one of the basic functions of the large-scale map. The accuracy of analytical calculation of the area of a geometric figure is the function of its size and shape and it directly depends on the accuracy of determination of position of its vertices.

The precision of notation of areas is important since in the case of determination of market values of real estates, it is the key attribute and the transaction price is directly proportional to the parcel area. Therefore, acquisition of information concerning the size of real estates, characterized by the high level of reliability, follows the widely understood interests of real estate owners.

Positions of border points should be determined with the accuracy of 0.01–0.03 m. Determination of vertices using the RTK GNSS technology is particularly attractive with respect to accuracy and economic issues [11].

According to the author's opinion, further investigations are required in order to establish effective procedures of explicit determination of the permissible differences of area determination basing on coordinates – with respect to surface object of arbitrary shapes. Such procedures are needed to determine object areas with the assumed level of reliability [22], without the necessity to commence changes in cadastral registers, resulting from discrepancies between areas (within the permissible scope, resulting from random measuring errors) and to meet the conditions of implementation of the basic function of the cadastral database, i.e. maintain the update and reliable information on real estates [23].

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